# Multi-Agent Based Federated Control of Large-Scale Systems with Application to Ship Roll Control

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***Abstract*—Large-scale systems refer to systems that consist of many interconnected local systems. Conventional centralized control schemes are not suitable for such large-scale systems because of their complex local and global dynamic behavior as well as computational difficulties. This paper introduces the general framework of an agent-based federated control motivated by the political structure where partially self-governing states are united by a federal government. Likewise, a multi-agent based federated control system is composed of local autonomous subsystems (agent-based controllers) that cooperate to provide an overall (large-scale) system behavior. In this concept, each agent has partial observations of the state of other agents and executes the local control law correspondingly to satisfy the performance requirements at the overall system level. Preliminary results are presented on the general architecture of multi-agent federated control for local and global connective stability.**

## I. INTRODUCTION

System control is a growing research area with many innovative techniques being introduced. During the past decade, most research work focused on systems with first or second order linear or nonlinear dynamics [1] [2] [3]. Today's systems are greatly advanced and complex, and demand more flexibility on system configurations and control schemes. Current control research is focused on systems with complex dynamics that are often classified as large-scale systems. Large-scale systems refer to systems that consist of several interconnected local systems, which may be coupled in some sort of configuration for a common performance goal. A large scale dynamic system is characterized by three factors:

1) High dimensionality of state variables
2) High complexity of computation
3) High search complexity of the action space

Systems of this kind appear, for example, in electric power systems, modern industrial applications, robotic systems, communication networks, economic systems and traffic networks.

When designing a large-scale system, it is often more effective to consider the system as a collection of several subsystems, and then to design each subsystem and their relationships [4]. To analyze the stability of large systems, one approach is to determine if the smaller systems are input-output reachable and controllable. Control design of complex large-scale systems cannot be done using conventional centralized techniques because of complexity of their dynamic behavior as well as computational difficulties. In addition, implementation of traditional centralized control is also problematic since any small change in the system dynamics will require a complete redesign of the centralized controller. It is clear that centralized control paradigms cannot meet the challenges of global performance requirements and stability.

For control of large-scale interconnected systems, it is typical to have some form of decentralized control architecture. These large-scale control systems have several local controllers, which observe only local outputs and control only local inputs, according to the performance requirements of the local systems. Each local controller is only involved in the local system control operation, yet the local controllers may be interconnected on some level of the global system. Therefore, the performance of each local controller effects the overall system performance and stability. But, these traditional decentralized methods can have flexibility and scalability issues.

New theoretical and application issues are arising as a result of current trends in distributed control, such as multi-agent based control. One of the key objectives of agent-based control is to use the decentralized approach but guarantee local and global closed loop stability, while reducing the control systems' computational load that is related to a centralized approach. Sycara [5] defined a multi-agent system as a collection of autonomous agents that interact with each other and their environment for the purpose of accomplishing a common objective. In other words, a multi-agent system is "a loosely coupled network of problem solvers (agents) that interact to solve problems that are beyond the individual capabilities or knowledge of each problem solver" [6]. Examples include devices or entities governed by software agents, such as a group of robots, cars in traffic on the road, or automated equipment pieces in an assembly line. Multi-agent systems can manifest self-organization and complex behaviors even when the individual strategies of all their agents are simple.

In the past few years, multi-agent concept studies are generally focused on the development of decentralized control laws in order to reach a global objective

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[7] [8] [9] [10] [11] [12] [13]. A common technique is for agreement to be reached between agents through "consensus" regarding a certain quantity of interest that depends on the state of all agents [14]. Many researchers find that agent-based control is suitable for decentralized cooperative control. However, common agent-based control methods may be too simple to explain sub-system dynamic behaviors. Agent-based control normally models each sub-system as a particle and ignores the sub-system (local) dynamics.

## II. MULTI-AGENT BASED FEDERATED CONTROL ARCHITECTURE

This paper investigates a new concept of control: federated control, motivated by the political structure of a federal government. Each state of the union maintains some level of political autonomy and at the same time must comply with the federal government policies. Likewise, a multi-agent based federated control system is composed of local autonomous entities (agent-based controllers) that cooperate to provide an overall (a large-scale) system behavior. Each agent based controller maintains its own local stability and has partial observations of the state of other agents. The agents execute their local control laws in order to satisfy the performance requirements at the overall system level.

In this concept, each agent represents an individual independent complete dynamic process with its own control law, which is interconnected with its appropriate neighbor agent(s). Interconnection of the agents through a communication network forms the federation in a distributed large-scale complex system. Fig. 1 illustrates this concept.

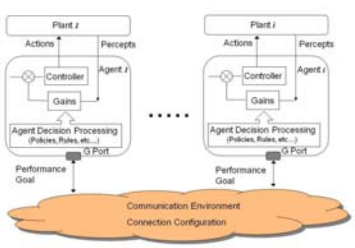


Fig. 1. Multi-agent Federated Control System

The concept of federated control with multi-agents provides the capability to revolutionize the system requirements dynamically. It enhances the overall system performance robustness. Agents will negotiate their local and global stability constraints following a federated "goal." The global "Goal" at the federal level is communicated to the agents via the communication network. The attributes of this "Goal" could be an output-input function, state equality constraints, input inequality constraints, cost function, or control logic, etc. Each agent makes its control decision independently and adjusts its controller accordingly upon receiving the federal "Goal" request and the state information from other agents at the local level. The agent could reject the "Goal" request if its local stability is threatened. Agents are self-aware, self-interested and self-protective to maintain their own stability, but the overall system stability will be satisfied as well.

## III. MULTI-AGENT BASED FEDERATED CONTROL FORMULATION

The control goal is to design a federated, multi-agent based controller for each local system and guarantee connective stability of the overall system. The concept of connective stability requires that the system remains stable in the sense of Lyapunov under structural perturbation [15], whereby local systems are disconnected and connected again in unpredictable ways during operation. The objective of multi-agent based federated control and the stability analysis is to prove that there exist vector Lyapunov functions for each of the individual local systems and that the vector sum of these Lyapunov functions is a Lyapunov function for the overall connective system. In the large-scale interconnected system, a vector Lyapunov function provides an extremely flexible stability analysis framework since each sub-system of the vector Lyapunov function can satisfy less strict requirements compared to single scalar Lyapunov functions. Hence, it is preferred to use a vector Lyapunov function to develop the control design of a large-scale interconnected system and prove stability [16] [17] [18] [19].

An interconnected large-scale system can be mathematically represented by

$$\mathbf{S}: \quad \begin{aligned} \dot{x}_i &= f_i(t,x_i,u_i) + h_i(t,x) \quad i \in \{1,\cdots,N\} \\ y_i &= C_i x_i \end{aligned} \tag{1}$$

where $N$ is the number of independent subsystems with local dynamics $S_i$: $\dot{x}_i = f_i(t,x_i,u_i)$ that are interconnected through $h_i = h_i(t,\overline{e}_{i1}x_1,\overline{e}_{i2}x_2,\cdots,\overline{e}_{iN}x_N)$. The elements of the fundamental interconnection matrix are defined as $\overline{E} = (\overline{e}_{ij})$, and

$$\overline{e}_{ij} = \begin{cases} 0, & x_j \quad \text{does not occur in} \quad h_i(t,x) \\ 1, & x_j \quad \text{occurs in} \quad h_i(t,x) \end{cases} \tag{2}$$

The elements $e_{ij}$ represent the connectivity between the individual subsystems (local agents). As stated in [15], the system is connectively stable if it is stable in the Sense of Lyapunov for all possible interconnection matrices $E = (e_{ij})$ (denoted $E \in \overline{E}$).

From the perspective of multi-agent federated control, the subsystem connectivity configuration is controlled by an agent which also manages the federated control law to maintain the local and global stability. The subsystem connectivity may be established by an agent for the purpose of implementing a control law to meet the performance goals at the federated level. Each local agent observes its own state and the output states from its connected neighbors and determines the maximum tolerance related to interconnection to its neighbors. Fig. 2 shows several key components and the structure of an agent controller. The system shown in Fig. 1 and Fig. 2 can be constructed as

$$\mathbf{S_i}: \quad \dot{x}_i = A_i x_i + B_i u_i \qquad i \in \{1, \cdots, N\} \tag{3}$$
$$y_i = C_i x_i$$

Note that each subsystem is independent and can operate as a stand-alone system unless there is a need for subsystem interconnection to meet federal goals. The control can be defined as $u_i = u_i^{\text{Local Control}} + u_i^{\text{Agent Control}}$ where the Local Control is determined to maintain the local subsystem stability and other performance requirements, whereas federal performance requirements are guaranteed by the agent control. The agent controller, $u^{\text{Agent Control}}$, is the decision processor at the federated level as illustrated in Fig. 2.

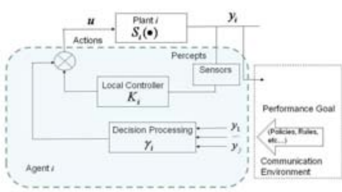


Fig. 2. Structure of an Agent

The overall system is composed of the team of agents which work together to provide a flexible framework to achieve total system stability. A general form of the $i$-th agent controller to achieve the federated control performance can be considered as

$$u_i^{AgentControl} = \underbrace{\gamma_2 \sum_{j \in N} e_{ij}(y_i - y_j)}_{\text{Consensus term}} + \underbrace{\gamma_1(-k_i(x_i^{desired} - x_i))}_{\text{Federated term}} \tag{4}$$

where the $\gamma$ terms are the weights. In equation (4), the agent selfishness is represented in the consensus term where each agent determines its interconnection strength to its neighbors, defined by the weight $\gamma_2$. The agents can disconnect the interrelation with their unstable neighbors when their local stability is threatened, defined by $e_{ij}$. Finally, the overall system performance goals are defined in the federated term. An example of the federated term is a trajectory for the group of agents to follow.

In a multi-agent federated control system, the dynamics of each local system can be different. The agent controller $u_i$ is not identical for every agent due to the inclusion of the federated and consensus terms. Therefore, the controller performance of each local agent is self-adjusted to satisfy local stability while following the federal guideline, as well as to achieve the overall system stability and performance at the federal level.

## IV. STABILITY OF LARGE-SCALE SYSTEMS

This section demonstrates how a simple control law for each agent can guarantee stability of the overall system. As an example, consider the interconnected system shown in Fig. 3 in which several subsystems are interconnected, forming a chain. The control input of each local system includes the output state of its neighboring system through a gain $\gamma$. The goal is to use multi-agent federated control to maintain global stability of the complete system.

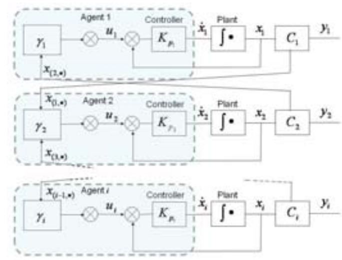


Fig. 3. An Input/Output Interconnected System via Multi-agents

If the subsystems are linear, the complete system can be represented as

$$\mathbf{S}: \quad \dot{x}_i = A_i x_i + B_i u_i + \gamma_i \sum_{j=1}^{N} e_{ij} A_{ij} x_j \quad i \in \{1, \cdots, N\} \tag{5}$$

where $e_{ij}$ is the interconnection matrix and $\gamma_i$ is the consensus connective strength.

Stability analysis is based on the concept of vector Lyapunov functions developed by [20]. The vector Lyapunov function for each individual system is defined as

$$V_i(x_i) = (x_i^T H_i x_i) \tag{6}$$

where $H_i$ is a positive definite matrix. The function $V_i(x_i)$ satisfies the Lipschitz condition with an existing Lipschitz constant $\lambda_{max}^{\frac{1}{2}}(H_i)$. Then, the system $\mathbf{S}$ is connectively stable if the matrix $\overline{W}$ is an aggregate matrix or M-matrix.(i.e. real symmetric positive definite matrix)

$$\overline{W} = (\overline{w}_{ij}) = \begin{cases} \frac{\lambda_{min}(G_i)}{2\lambda_{max}(H_i)} - \overline{e}_{ii}\lambda_{max}^{\frac{1}{2}}(A_{ii}^T A_{ii}), & i = j \\ -\overline{e}_{ij}\lambda_{max}^{\frac{1}{2}}(A_{ij}^T A_{ij}), & i \neq j \end{cases} \tag{7}$$

Here $G_i$ is a symmetric positive matrix and satisfies the Lyapunov matrix equation:

$$A_i^T H_i + H_i A_i^T + G_i = 0 \tag{8}$$

It is also known that the larger the diagonal elements $\overline{w}_{ii}$ and the smaller the off-diagonal elements $\overline{w}_{ij}$, the better the opportunity to drive $\overline{W}$ to be an M-matrix. Clearly, the stability matrix $G_i$ in equation (8) needs to be chosen to maximize the ratio of $\frac{\lambda_{min}(G_i)}{\lambda_{max}(H_i)}$. This ratio is the estimation of the degree of stability for each large-scale system $S_i$.

*A. Chain of Interconnected Integrators*

For the particular linear input-output connected system as shown in Fig. 3, the following analysis shows that the agents play the major role in connecting and maintaining stability of the overall system. For simplicity of presentation, it is assumed that the subsystem plant is an integrator with a proportional feed forward gain and a unity feedback loop. It is also assumed that the connective strength $\gamma$ is identical for every subsystem. Then the dynamics of each subsystem could be described as

$$\begin{aligned} \dot{x}_i &= -k_p x_i + k_p u_i \\ y_i &= C x_i \qquad \text{where } C = I \end{aligned} \tag{9}$$

where $x_i$ is the state of the $i$-th system, $u_i$ is the control input, and $y$ is the output of the subsystem.

It is assumed that each agent needs to determine the connective strength which it can interrelate to its neighbors. It is noted that the potential term and the federated term are neglected for simplicity in this example. Then, the agent controller is simple consensus control

$$u_i = \gamma \sum_{j \in N} (y_i - y_j) \tag{10}$$

For this particular configuration, the input of each subsystem is

$$\begin{aligned} u_1 &= \gamma x_2 \\ u_i &= \gamma(x_{i-1} + x_{i+1}) \qquad i \in \{2, \cdots, N-1\} \\ u_N &= \gamma x_{N-1} \end{aligned} \tag{11}$$

where $\gamma$ is the connective strength to be determined by each agent. Then the generalized system equation is

$$\mathbf{S:} \quad \dot{x}_i = \hat{A}_i x_i + \gamma \sum_{j=1}^{N} e_{ij} \hat{A}_{ij} x_j \quad i \in \{1, \cdots, N\} \tag{12}$$

where $\hat{A}_i$ donates a closed loop subsystem. In the above equation, $e_{ij}$ is the agent communication connectivity switch which allows the agent to maintain interconnection with neighboring agents. The system matrix is $A_i = -k_p$ for the system of (9). The expanded system equation is

$$\begin{aligned} \dot{x}_1 &= -k_p x_1 + e_{11}\gamma k_p x_1 + e_{12}\gamma k_p x_2 + \cdots \\ \dot{x}_2 &= -k_p x_2 + e_{21}\gamma k_p x_1 + e_{22}\gamma k_p x_2 + e_{23}\gamma k_p x_3 + \cdots \\ &\cdots \\ \dot{x}_N &= -k_p x_N + e_{N1}\gamma k_p x_1 + e_{N2}\gamma k_p x_2 + e_{N3}\gamma k_p x_3 + \cdots \end{aligned} \tag{13}$$

Since the multi-agents manage the stability and the connectivity of each subsystem, the $W$ matrix can be constructed. In addition, the gain $k_p$, connective strength $\gamma$ and connectivity parameter $e_{ij}$ need to be determined. For the large scale system of (13), the test matrix becomes

$$W = (W_{ij}) = \begin{bmatrix} k_p & -\gamma k_p & 0 & 0 & 0 & \cdots & 0 \\ -\gamma k_p & k_p & -\gamma k_p & 0 & 0 & \cdots & 0 \\ 0 & -\gamma k_p & k_p & -\gamma k_p & 0 & \cdots & 0 \\ 0 & 0 & -\gamma k_p & k_p & -\gamma k_p & & 0 \\ \vdots & & & & & \ddots & \vdots \\ \vdots & & & & -\gamma k_p & k_p & -\gamma k_p \\ 0 & \cdots & & & \cdots & -\gamma k_p & k_p \end{bmatrix} \tag{14}$$

In what follows, we shows that the maximum tolerance factor $\gamma$ reaches a limit of $\gamma = \frac{1}{2}$ for this particular large system exploiting the multi-agent based federated control method.

**Proof:** Equation (14) could be rewritten as

$$\begin{aligned} W &= k_p \mathbf{I} + \gamma k_p \mathscr{E} \\ &= k_p(\mathbf{I} + \gamma \mathscr{E}) \end{aligned} \tag{15}$$

$$\text{where} \quad \mathscr{E} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & \cdots & 0 \\ -1 & 0 & -1 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & -1 & 0 & -1 & & 0 \\ \vdots & & & & & \ddots & \vdots \\ \vdots & & & & -1 & 0 & -1 \\ 0 & \cdots & & & \cdots & -1 & 0 \end{bmatrix} \tag{16}$$

Given an arbitrary vector $\zeta$ satisfying $-\|\mathscr{E}\|\|\zeta\|^2 \le \langle \mathscr{E}\zeta, \zeta \rangle \le \|\mathscr{E}\|\|\zeta\|^2$, then

$$\begin{aligned} \langle W\xi, \xi \rangle = \langle (\mathbf{I} + \gamma\mathscr{E})\xi, \xi \rangle &\equiv \|\xi\|^2 + \gamma \langle \mathscr{E}\xi, \xi \rangle \\ &\ge \|\xi\|^2 - \gamma \|\mathscr{E}\|\|\xi\|^2 \\ &\equiv (1 - \gamma\|\mathscr{E}\|)\|\xi\|^2 \end{aligned}$$

In order to assure $W$ to be positive definite, then $1 - \gamma\|\mathscr{E}\| > 0$ must be satisfied. That gives

$$\gamma < \frac{1}{\|\mathscr{E}\|} = \begin{cases} \frac{1}{\max_i(\sum_j |\mathscr{E}_{ij}|)} = \frac{1}{|-2|}, & \text{finding max in row} \\ \quad \text{or} \\ \frac{1}{\max_j(\sum_i |\mathscr{E}_{ij}|)} = \frac{1}{|-2|}, & \text{finding max in column} \end{cases}$$

Therefore the connective strength $\gamma$ should be selected less than 0.5 to guarantee the global and local stability for the large-scale system. Note also that this result holds for arbitrary interconnection $e_{ij}$ as subsystems enter or leave the federation. ■

From the perspective of multi-agent concepts, each subsystem shown in Fig. 3 is controlled by an agent with its own control law as defined in (11). The subsystem stability is determined by the forward gain coefficient $k_p > 0$, and $\gamma$ is the connective strength determined by each agent. The objective at the federal level is to maintain global stability of the entire interconnection.

Initial investigations show that the system is stable at the federated level if the connective strength $\gamma$ remains within the limit of $0 < \gamma < \frac{1}{2}$ for large $N$. In particular, $\gamma = 1$ for an interconnected federation of $N = 2$, $\gamma = 0.707$ for $N = 3$ and $\gamma = 0.618$ for $N = 4$. Therefore, for this particular large-scale system using multi-agent based federated control, the connective strength $\gamma$ should be selected less than 0.5 to guarantee stability of the overall federated system and each local subsystem for an arbitrary interconnection of $N$ subsystems.

One of the advantages in connective strength $\gamma$ computation is that the agent computes $\gamma$ only according to its own endurance level to the external influence. The agent does not need to know the system information of the neighbor agent.

## V. SIMULATION EXAMPLES

As an example, a numerical simulation is given to illustrate the effectiveness of the multi-agent based federated control. This system consists of two ships cruising parallel to each other on the same heading. The objective is that each ship's control agent manages the stability of its own ship roll motion and observes the roll state for the other ship simultaneously during the sea state in order to achieve global stability of the entire system. There is no physical connection between the two ships. The system state variables are the ship roll motion angle $\theta$ and the angular velocity $\dot{\theta}$. Let $S_i$, $i = 1,2$ represent the mathematical model of the two ships as shown in Fig. 4. The ship models are described as

$$\mathbf{S_1}_{open\text{-}loop}: \quad \dot{x}_1 = \begin{bmatrix} 0 & 1 \\ -0.0288 & 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1 \qquad (17)$$

$$\mathbf{S_2}_{open\text{-}loop}: \quad \dot{x}_2 = \begin{bmatrix} 0 & 1 \\ -0.0029 & 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_2 \qquad (18)$$

where $x_1 = \begin{bmatrix} \theta_1 & \dot{\theta}_1 \end{bmatrix}^T$, $x_2 = \begin{bmatrix} \theta_2 & \dot{\theta}_2 \end{bmatrix}^T$

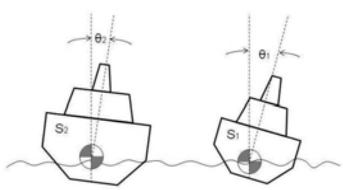


Fig. 4. Ship Roll Motion Control

For each ship $S_i$, $i = 1,2$ the control agent implements a roll motion controller for local stability. The closed-loop systems (17) and (18) are designed so that the eigenvalues of the closed-loop systems $S_1$ and $S_2$ are selected as $[-6.725 \pm 6.2832i]$ and $[-5.1065 \pm 6.2832i]$, then both systems are individually locally stable. It is assumed that each ship partially observes the state of the other ship, so that it receives the neighbor agent roll sensor data via the communication media. Each agent maintains an interconnection with the neighboring agent using connective strength $\gamma_1$ and $\gamma_2$ defined by the closed loop equations

$\mathbf{S_1}_{closed\text{-}loop\text{-}interconnected}$ :

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ -84.7035 & -13.4499 \end{bmatrix} x_1 + \begin{bmatrix} 0 & \gamma_1 \\ \gamma_1 & 0 \end{bmatrix} x_2 \qquad (19)$$

$\mathbf{S_2}_{closed\text{-}loop\text{-}interconnected}$ :

$$\dot{x}_2 = \begin{bmatrix} 0 & 1 \\ -65.5551 & -10.2131 \end{bmatrix} x_2 + \begin{bmatrix} 0 & \gamma_2 \\ \gamma_2 & 0 \end{bmatrix} x_1 \qquad (20)$$

Each ship's control agent (19) and (20) takes into consideration its neighbor agent's state. In the event of an unstable agent, that agent will be isolated from the multi-agent based connection network and the large-scale system retains its global stability. It is obvious that appropriately determining the connective strength $\gamma$ for every agent is the key factor to achieving the system global stability. In other words, the agent needs to determine its connective strength $\gamma$ prior to making a connection to the other systems in order to maintain its connective stability and the overall system stability as well.

The following illustrates the connective strength $\gamma_1$ computation procedure for the system (19). Suppose the Lyapunov function for system (19) is defined as

$$V(x_1) = x_1^T H x_1 \qquad (21)$$

where $H$ is a positive definite matrix. Then the system $S_1$ will be connectively stable based on the determination of $\gamma_1 = max\{\frac{\lambda_{min}(G)}{2\lambda_{max}(H)}\}$, subject to the Lyapunov matrix equation revised as

$$A^T HI + IHA^T = -G \qquad (22)$$

From the properties of Kronecker product of matrices, it follows that

$$(I \otimes A)\hat{H} \quad + \quad (A \otimes I)\hat{H} = -\hat{G} \qquad (23)$$
$$\hat{H} \quad = \quad -M^{-1}\hat{G} \qquad (24)$$

where $M = I \otimes A + A \otimes I$, and $\hat{H}$ is the vectorized form of the matrix $H$.

Hence, $\hat{H}$ equals to $[0.3510, -1.500, -1.500, 9.5581]^T$ when $G$ was selected to be equal to $3I$. This implies that the connective strength is $\gamma_1 = 0.3062$. Using the same computation procedure, $\gamma_2 = 0.2997$ is obtained. Simulation results illustrate that both ship roll control agents successfully stabilize the ship roll motion when an initial disturbance is presented as shown in Fig. 5.

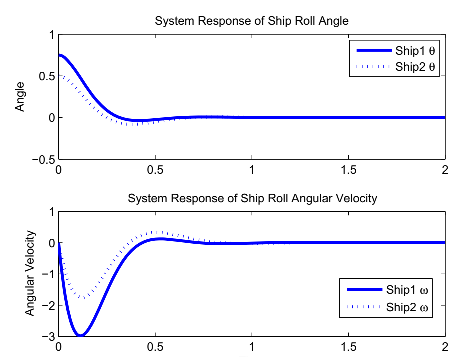


Fig. 5. Ship Motion $\theta$ and $\omega$ Control

Furthermore, additional simulation results show that since the connective strength was properly chosen, the ship 1 roll motion stabilizing performance was minimally impacted by the roll oscillation of ship 2. These results are shown in Fig. 6. However, if the connective strength are improperly computed and the two subsystems are connected through these wrongly computed $\gamma$s, then the overall connected system will be unstable. Fig. 7 illustrates that both ships are not converging due to some erroneous $\gamma$ computation. It is also worth stating that the stability of the subsystems $S_1$ and $S_2$ implies stability of the overall large-scale system. For instance, if there is an unstable subsystem to be connected to a stable neighbor system, then the stable neighbor system will become unstable and the large-scale system will be cascade-effect unstable. Therefore the condition of overall system connective stable is based on the stability of each connected subsystems and properly choosing the connective strength factor for every connected subsystem.

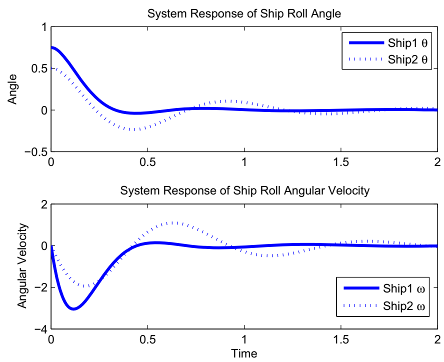


Fig. 6. Ship Motion $\theta$ and $\omega$ Control (Ship 2 Oscillation)

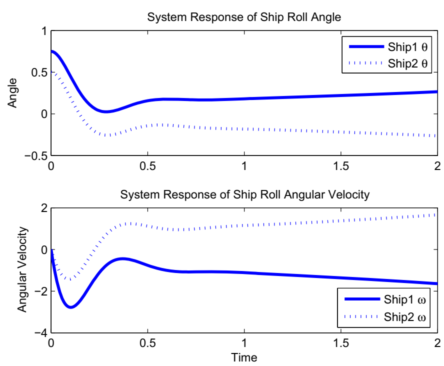


Fig. 7. Ship Motion $\theta$ and $\omega$ Control (Improperly Computed $\gamma$)

## VI. CONCLUSIONS AND FUTURE WORK

The connective stabilization problem for the multi-agent based control of large-scale systems, which is constituted by connecting local systems one after another serially, was studied in this paper. Use of multi-agent control as the system connective switch to tie local systems into a large-scale system was introduced. Large-scale system stability through a multi-agent based controller is obtained by taking vector Lyapunov functions and computing the appropriate agent connective strength. The method of the appropriate agent connective strength for the multi-agent based connective stabilization controller was developed. Most importantly, the computation of the appropriate agent connective strength can be executed at the local agent level, requiring minimal information about the rest of the system. Furthermore, this method can be expended to achieve system connectivity reconfiguration without changing the control laws of the original system. A numerical example also showed the effectiveness of this developed method. The featured work could be extended to utilize multi-agent based federated control to perform ship collision avoidance.

## VII. ACKNOWLEDGMENTS

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## REFERENCES

[1] R. Dorf and R. Bishop, *Modern Control Systems – [10th ed.]*.
[2] K. Ogata, *Discrete-time Control Systems – [2nd ed.]*.
[3] H. Khalil, *Nonlinear Systems – [2nd ed.]*.
[4] S.Kumagai and T.Miyamoto, "An agent net approach to autonomous distributed systems," Presented at the IEEE Int. Conf. on Systems, Man & Cybernetics., 1996.
[5] K. P. Sycara, "Intelligent agents," *AI magazine*, vol. 19, no. 2, Nov. 1998.
[6] E. Durfee and V. Lesser, "Negotiating task decomposition and allocation using partial global planning," *Distributed Artificial Intelligence*, vol. 2, pp. 229–244.
[7] A. Jadbabaie, J. Lin, and A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 988–1001, June 2003.
[8] N. L. P. Ogren and E. Fiorelli, "Cooperative control of mobile sensing networks: Adaptive gradient climbing in a distributed environment," *IEEE Transactions on Automatic Control*, vol. 49, no. 8, pp. 1292–1302, Aug. 2004.
[9] J. Cortes, S. Martinez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks," *IEEE Transactions on Robotics and Automation*, vol. 20, no. 2, pp. 243–255, Feb. 2004.
[10] R. O. Saber, "Flocking in multiagent dynamic systems: Algorithms and theory," *IEEE Transactions on Automatic Control*, vol. 51, no. 3, Mar. 2006.
[11] E. Justh and P. Krishnaprasad, "Equilibria and steering laws for planar formations," *Systems and Control Letters*, vol. 52, no. 1, pp. 25–38, May 2004.
[12] F. Cucker and S. Smale. (2010) Emergent behavior in flocks. [Online]. Available: http://ttic.uchicago.edu/~smale/papers/flock.pdf
[13] J. Cortes, S. Martinez, and F. Bullo, "Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions," *IEEE Transactions on Automatic Control*, vol. 51, no. 8, pp. 1289–1298, Aug. 2006.
[14] R. O. Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," in *Proceedings of the IEEE*, vol. 95, no. 1.
[15] D. Siljak and M. Ikeda, "Stability of reduced-order models via vector liapunov functions," presented at the American Control Conference, 1987, pp. 482–489.
[16] V. Lakshmikantham and S. Leela, *Differential and Integral Inequalities*. Academic Press, 1969, vol. 55-1.
[17] V. Lakshmikantham, V. M. Matrosov, and S. Sivasundaram, *Vector Lyapunov Functions and Stability Analysis of Nonlinear Systems*. Dordrecht, Netherlands: Kluwer Publishers, 1991.
[18] D. D.Siljak, *Large Scale Dynamic Systems: Stability and Structure*. New York, USA: North Holland, 1978.
[19] V. M. Matrosov, "Methods of vector lyapunov functions of interconnected systems with distributed parameters," *Avtomatika i Telemekhanika*, vol. 33, pp. 63–75, 1972.
[20] D. D. Siljak, *Decentralized Control of Complex Systems*. Academic Press, 1991, ch. 2.