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Cooperative Federated Control with Application to Tracking Control

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***Abstract*—Large-scale systems refer to systems that consist of many interdependent local systems. Conventional centralized control schemes are not suitable for such large-scale systems due to global dynamic behavior complexity and computational difficulties. This paper introduces a general framework of agent-based federated control motivated by the political structure where partially self-governing states are united by a federal government. Likewise, a multi-agent based federated control system is composed of local autonomous subsystems that cooperate to provide an overall system behavior. Formation stability and trajectory tracking control of a group of autonomous agents is designed based on this concept. Decentralized cooperative federated controllers are developed for each individual agent. Simulations show that the overall system is globally asymptotically stable to track a time-varying trajectory with a pre-designated formation for a group of nonlinear dynamic agents.**

***Index Terms*—Multi-agent control, federated systems, large-scale systems, stability**

I. Introduction

System control is a growing research area with many innovative techniques being introduced. During the past decade, most research work focused on systems with first or second order linear or nonlinear dynamics [1] [2] [3]. Today's systems are greatly advanced and complex, and demand more flexibility on system configurations and control schemes. Current control research is focused on systems with complex dynamics that are often classified as large-scale systems. Large-scale systems refer to systems that consist of several interconnected local systems, which may be coupled in some sort of configuration for a common performance goal. A large scale dynamic system is typically operated under high dimensionality of state variables and requires high complexity of computation. When designing a large-scale system, it is often more effective to consider the system as a collection of several subsystems, and then to design each subsystem and their relationships [4]. To analyze the stability of large systems, one approach is to determine if the smaller systems are input-output reachable and controllable. Control design of complex large-scale systems cannot be done using conventional centralized techniques because of complexity of their dynamic behavior as well as computational difficulties. In addition, implementation of traditional centralized control is also problematic since any small change in the system dynamics will require a complete redesign of the centralized controller. It is clear that centralized control paradigms cannot meet the challenges of global performance requirements and stability.

For control of large-scale interconnected systems, it is typical to have some form of decentralized control architecture. These large-scale control systems have several local controllers, which observe only local outputs and control only local inputs, according to the performance requirements of the local systems. Each local controller is only involved in the local system control operation, yet the local controllers may be interconnected on some level of the global system. Therefore, the performance of each local controller effects the overall system performance and stability. But, these traditional decentralized methods can have flexibility and scalability issues.

With the rapid growth in high performance computing and communications technology, new theoretical and application issues are arising as a result of current trends in distributed control, such as multi-agent based control. One of the key objectives of agent-based control is to use the decentralized approach but guarantee local and global closed loop stability, while reducing the control systems' computational load that is related to a centralized approach. Sycara [5] defined "a multi-agent system as a collection of autonomous agents that interact with each other and their environment for the purpose of accomplishing a common objective". In other words, a multi-agent system is "a loosely coupled network of problem solvers (agents) that interact to solve problems that are beyond the individual capabilities or knowledge of each problem solver" [6]. Multi-agent systems can manifest self-organization and complex behaviors even when the individual strategies of all their agents are simple [7] [8] [9]. The decentralized control allocates the processing power among agents and gains the complex behavior out of simplistic agents. Benefits of multi-agent cooperative control include improving robustness, reliability, intelligence, performance, and flexibility of large-scale systems.

In the past few years, multi-agent concept studies are generally focused on the development of decentralized control laws in order to reach a global objective [10] [11] [12] [13] [14] [15] [16]. A common technique is for agreement to be reached between agents through "consensus" regarding a certain quantity of interest that depends on the state of all agents [17]. Many researchers find that agent-based control is suitable for decentralized cooperative control. However, common agent-based control methods may be too simple to explain sub-system dynamic behaviors. Agent-based control normally models each sub-system as a particle and ignores

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the sub-system (local) dynamics. An important study in multi-agent systems is the development of cooperative behavior between agents that have a shared goal. Multi-agent formation and cooperative control can play an important role in military applications. Cooperative control of multiple aerial, surface, or underwater vehicles has important applications for the U.S. Navy. These applications include fleet navigation, transporting loads, search and rescue operations, etc. Cooperative control systems also appear often in nature. Examples include flocks of birds, schools of fish, and cooperative ant colonies. Recently, there has been close cooperation between engineers and biologists for better modeling and understanding of the complicated behaviors of creatures in nature [10]. Connective stability has been also considered by many researchers [18].

In this paper, decentralized cooperative federated controllers are developed for each individual agent. Simulations show that the overall system is globally asymptotically stable when tracking a time-varying trajectory with a pre-designated formation for a group of nonlinear dynamic agents. The paper is organized as follows: Section II introduces the basic notion of federated control. Control system formulation for federated control is presented in Section III and its stability in Section IV. Section V presents simulation results that illustrate the concept. The paper is concluded with some final remarks in Section VI.

II. Federated Control Architecture

This paper investigates a new concept of control: *federated control,* motivated by the political structure of a federal government. Each state of the union maintains some level of political autonomy and at the same time must comply with the federal government policies. Likewise, a multi-agent based federated control system is composed of local autonomous entities (agent-based controllers) that cooperate to provide an overall (a large-scale) system behavior. Each agent based controller maintains its own local stability and has partial observations of the state of other agents. The agents execute their local control laws in order to satisfy the performance requirements at the overall system level.

In this concept, each agent represents an individual independent complete dynamic process with its own control law, which is interconnected with its appropriate neighbor agent(s). Interconnection of the agents through a communication network forms the federation in a distributed large-scale complex system. Fig. 1 illustrates this concept.

The novel concept of federated control with multi-agents provides the capability to revolutionize the system requirements dynamically. It enhances the overall system performance robustness. Agents will negotiate their local and global stability constraints following a federated *"Goal"*. The global *"Goal"* at the federal level is communicated to the agents via the communication network. The attributes of this *"Goal"* could be an output-input function, state equality constraints, input inequality constraints, cost function, or control logic, *etc*. Each agent makes its control decision independently and adjusts its controller accordingly upon receiving the federal *"Goal"* request and the state information from other agents

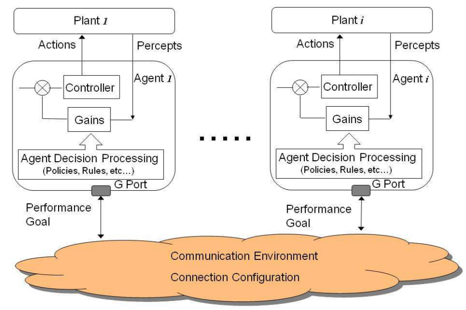

Fig. 1. Multi-agent Federated Control System

at the local level. The agent could reject the *"Goal"* request if its local stability is threatened. Agents are self-aware, self-interested and self-protective to maintain their own stability, but the overall system stability will be satisfied as well.

III. Federated Control Formulation

The control goal is to design a federated, multi-agent based controller for each local system and guarantee connective stability of the overall system. The concept of connective stability requires that the system remains stable in the sense of Lyapunov under structural perturbation [19], whereby local systems are disconnected and connected again in unpredictable ways during operation. The objective of multi-agent based federated control and the stability analysis is to prove that there exist vector Lyapunov functions for each of the individual local systems and that the vector sum of these Lyapunov functions is a Lyapunov function for the overall connective system. In the large-scale interconnected system, a vector Lyapunov function provides an extremely flexible stability analysis framework since each sub-system of the vector Lyapunov function can satisfy less strict requirements compared to single scalar Lyapunov functions. Hence, it is preferred to use a vector Lyapunov function to develop the control design of a large-scale interconnected system and prove stability [20] [21] [22] [23].

An interconnected system of agents can be mathematically represented by

$$\mathbf{S_i}: \quad \dot{x}_i = f_i(t, x_i) + u_i \quad i \in \{1, \cdots, N\} \tag{1}$$

where N is the number of independent agents with local dynamics S_i. The control input has two components defined as $u_i = u_i^{\text{Local Control}} + u_i^{\text{Agent Control}}$, where the *Local Control* is determined to maintain the local subsystem stability and other performance requirements, whereas federal performance requirements are guaranteed by the *Agent Control*. The agent controller, $u^{\text{Agent Control}}$, is the decision processing component at the federated level as illustrated in Fig. 2.

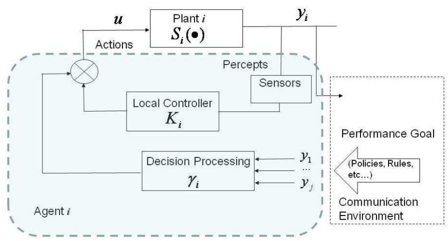

Fig. 2. Structure of an Agent

We assume that each agent is connected through $u^{\text{Agent Control}}$. The overall system is composed of the team of agents which work together to provide a flexible framework to achieve total system stability. A general form of the i-th agent controller to achieve the federated control performance is defined as

$$u_i^{AgentControl} = \underbrace{-\gamma_1 \sum_{j \in N} a_{ij}(x_i - x_j)}_{\text{Consensus term}} \underbrace{-\gamma_2 a_{i*}(x_i - x_*)}_{\text{Federated term}} \quad (2)$$

where $\gamma_{1,2}$ are the interconnection strengths, and x_* is the federated policy state. The agents can use the consensus term to determine whether or not to connect to the neighbors

$$a_{ij} = \begin{cases} 1, & \text{system } \mathbf{S}_i \text{ is connected to system } \mathbf{S}_j \\ 0, & \text{system } \mathbf{S}_i \text{ is not connected to system } \mathbf{S}_j \end{cases} \quad (3)$$

The federated goal is a self-contained system that does not require state information from other agent. Its dynamics are represented by

$$\mathbf{S}_* : \quad \dot{x}_* = f(t, x_*) \quad (4)$$

and all agents are expected to obey the federated goal. In this context, the trajectory x_* is the desired performance goal at the federated level.

The overall system performance goals are achieved by the federated term. In this paper, *"follow the trajectory x_*"* is considered to be a federated goal. Also note that agents receive the federated goal through the interconnection a_{i*} and pass it to other agents through consensus. Connection to the federated goal is defined by

$$a_{i*} = \begin{cases} 1, & \text{system } \mathbf{S}_i \text{ follows the system } \mathbf{S}^* \\ 0, & \text{system } \mathbf{S}_i \text{ does not follow the system } \mathbf{S}^* \end{cases} \quad (5)$$

Although the overall system can be constructed by many simple agents, the controllers u_i for each agent are not identical due to the inclusion of the federated and consensus terms. Therefore, the examination of the system's interconnected stability under such multi-agent controllers must be conducted. The interconnection weights γ_1 and γ_2 must be determined to maintain each agent's performance while achieving federated goal.

IV. Stability Analysis

Many physical systems are nonlinear systems, thus analysis of nonlinear dynamics should be considered. The combination of sub-system connectivity topology and nonlinear dynamics is the major challenge. Stability analysis of continuous-time coupled nonlinear systems based on graph theory and discrete set-valued Lyapunov functions was developed by [24]. Lyapunov stability theory was also utilized to design cooperative control for nonlinear systems with a time-varying bi-directional communication network [25]. It is desired that all the outputs of the dynamical systems converge on consensus as fast as possible. The interaction coefficients, such as the weights on the edges and between the vertices in the multi-agent systems, are optimized in [26]. However, the optimization criteria commonly used are the asymptotic exponential speed of convergence or pre-defined potential functions; such approach might not be the sufficient pertinent criterion over all applications [27]. Therefore, we attempt to derive a general method to obtain the sub-system connective strength in order to maintain the overall system stability.

This section demonstrates how a simple control law for each agent can guarantee stability of the overall system. We consider a multi-agent system composed by N agents, moving on a plane. The dynamics of each agent i are given in (1). In this multi-agent system, the agents are connected to other agents and some of the agents also receive the federated goal. The goal is to maintain global stability of the complete system. The multi-agent system can be represented by a directed graph $G(t) = (V, E, A)$, consisting of a set of vertices, or nodes, denoted $V = \{n_1, n_2, \cdots n_N\}$ and indexed by the agents in the group, and a set of edges, $E = \{(n_i, n_j) \in V \times V : n_i \neq n_j\}$ containing an ordered set of distinct vertices that represent the neighboring relations. An edge $E = (n_i, n_j)$ means that agent j receives information from agent i in G, and the agent j will regulate its states based on its relationship to the states of agent i. A spatial adjacency matrix $A = A(G) = [a_{ij}] \in R^{n \times n}$ is a weight matrix defined as $a_{ij} = 1$, if $(i, j) \in N_i$ and $a_{ij} = 0$ otherwise. The degree matrix of G is a diagonal matrix $D = [d_i]$ with $d_i = \sum_{j \in N_i} a_{ij}$, and $N_i = N_i(G) = \{j \in \nu : (i, j) \in E_i\}$, where $a_{ij} = a_{ji}$ in an undirected graph and $a_{ij} \neq a_{ji}$ in a directed graph. The element number of set N_i is the communication neighbor set of agent i. It indicates the number of edges connected to agent i, denoted by d_i. The Laplacian for the directed graph G is $L = D - A$. It is also worthwhile to note that the Laplacian L is not only positive semi-definite but also has one zero eigenvalue [28].

In the federated control concept, the federal *"Goal"* is handled outside of the group of agents. For that reason, the federal policy handler is introduced as node $n+1$ (or $*$). One special property of this federal policy handler is that it is directly connected to some of the nodes in G and delivers the federal goal to these nodes. For the purpose of this stability analysis, the Laplacian of the federated control graph, G_{n+1}, is defined as

$$L_{n+1} = \begin{bmatrix} \sum_{j=*,1}^{n} a_{1j} & -a_{12} & \cdots & -a_{1n} & -a_{1*} \\ -a_{21} & \sum_{j=*,1}^{n} a_{2j} & \cdots & -a_{2n} & -a_{2*} \\ \vdots & \cdots & \ddots & \vdots & \vdots \\ -a_{n1} & -a_{n2} & \cdots & \sum_{j=*,1}^{n} a_{nj} & -a_{n*} \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \quad (6)$$

and can be partitioned as

$$L_{n+1} = \begin{bmatrix} H & a_* \\ 0 & 0 \end{bmatrix} \quad (7)$$

The Laplacian of the directed graph G_{n+1} has zero row sum and non-positive diagonal elements. Also L_{n+1} is a diagonally dominant matrix with zeros in the last row. Obviously, L_{n+1} has exactly one zero eigenvalue and all nonzero eigenvalues are in the open left half plane since the directed graph G_{n+1} has a spanning tree [29]. As a result, the matrix H has no zero eigenvalues. From the Gershgorin Disk theorem [30] [31], all the eigenvalues λ_H of H are located in the union of n disks as

$$\| \lambda_H - \sum_{j=0}^{n} a_{ij} \| \leq \sum_{j=1}^{n} a_{ij} \| = r_i \quad (8)$$

where $r_i := \sum_{\substack{j=1 \\ j\neq i}}^{n} | a_{ij} |,\ i = 1, \cdots, n.$

Since $a_{ij} \geq 0$, that implies $\sum_{\substack{j=1 \\ j\neq i}}^{n} | a_{ij} | = \sum_{\substack{j=1 \\ j\neq i}}^{n} a_{ij}$. The Gershgorin Disk theorem in (8) obtains

$$a_{i0} \leq Re(\lambda) \leq a_{i0} + 2\sum_{j=1}^{n} a_{ij} \quad (9)$$

Therefore, the $Re(\lambda_H) \geq 0$ as a result of (9), where there exists a symmetric positive definite $P \in \mathbb{R}^{n\times n}$ satisfying the Lyapunov function $PH + H'P$. Furthermore, it concludes that $\lambda_{min}(PH + H'P) > 0$.

Next, note that the nonlinear function $\dot{x}_i = f_i(t, x_i)$ satisfies the Lipschitz conditions

$$\begin{aligned} |f(t,x) - f(t,y)| &\leq \kappa|x - y| \\ |f(t,x)| &\leq \kappa(1 + |x|) \end{aligned} \quad (10)$$

which must hold at least locally in $\mathbb{R}^n$ so as to guarantee the existence of a solution of the system (1).

To analyze the federated control, let us define the agent tracking error as $\hat{x}_i = x_i - x_*$ in (2), where the desired trajectory x_* is the state of federated goal in (5). Then (1) with the controller u_i can be re-written as

$$\hat{x}_i = -\gamma\big(\sum_j a_{ij}(\hat{x}_i - \hat{x}_j) + a_{i*}\hat{x}_i\big) + f(t, x_i) - f(t, x_*) \quad (11)$$

where $\hat{x}_i \in \mathbb{R}^n$. For notational simplicity, define

$$\hat{x} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \vdots \\ \hat{x}_N \end{bmatrix} \quad (12)$$

$$F(t,\hat{x}) = \begin{bmatrix} f(t,x_1) - f(t,x_*) \\ f(t,x_2) - f(t,x_*) \\ \vdots \quad \vdots \\ f(t,x_N) - f(t,x_*) \end{bmatrix} \quad (13)$$

It follows then that the complete federated system and (1) can be represented as

$$\begin{aligned} \dot{\hat{x}} &= -\gamma\big(\sum_{j\in N_i} a_{ij}\big(L\otimes I_n)\hat{x} + (D\otimes I_n)\hat{x}\big) + F(t,\hat{x}) \\ &= -\gamma(H\otimes I_n)\hat{x} + F(t,\hat{x}) \end{aligned} \quad (14)$$

where $H = L + diag(a_{1*}, \cdots a_{n*})$.

The Lyapunov candidate for each individual system is defined as

$$V(\hat{x}) = \hat{x}'(P\otimes I_n)\hat{x} \quad (15)$$

Taking the time derivative of V and using the equation (14), we obtain

$$\begin{aligned} \dot{V} &= \hat{x}'(P\otimes I_n)\dot{\hat{x}} + \dot{\hat{x}}'(P\otimes I_n)\hat{x} \\ &= -\gamma\hat{x}'\big((H'\otimes I_n)(P\otimes I_n) \\ &\quad + (P\otimes I_n)(H\otimes I_n)\big)\hat{x} \\ &\quad + 2\hat{x}'(P\otimes I_n)F(\hat{x}) \end{aligned} \quad (16)$$

The matrix P can be chosen to satisfy the Lyapunov function $-PH - H'P = Q$, where P is a positive definite matrix and Q satisfies the Rayleigh-Ritz inequality for any symmetric matrix $\lambda_{\min}(H)\|x\|^2 \leq x'Hx \leq \lambda_{\max}(H)\|x\|^2$. Then it follows from above that

$$\begin{aligned} \dot{V} &= -\gamma\hat{x}'\big((PH + H'P)\otimes I_n)\big)\hat{x} + 2\hat{x}'(P\otimes I_n)F(\hat{x}) \\ &\leq -\gamma\lambda_{\min}(Q)\|\hat{x}\|^2 + 2\lambda_{\max}(P)\|\hat{x}\|\|F(\hat{x})\| \\ &\leq -\left(\gamma - \frac{2\kappa\lambda_{\max}(P)}{\lambda_{\min}(Q)}\right)\lambda_{\min}(Q)\|\hat{x}\|^2 \end{aligned} \quad (17)$$

where $\kappa > 0$ is the (local) Lipschitz constant satisfying (10). The complete system will be locally asymptotically stable if the agent connective strength γ is chosen to satisfy

$$\gamma > \frac{2k\lambda_{\max}(P)}{\lambda_{\min}(Q)} \quad (18)$$

V. Simulation Example

In this section, a simulation is given to show the effectiveness of the proposed consensus and trajectory-tracking federated control algorithm. The example multi-agent system is composed of 5 agents on a plane. Each agent is defined by

$$\mathbf{S_i}: \quad \dot{x}_i = a_i\cos(x_i) \quad i \in \{1, \cdots, N\} \quad (19)$$

The agents are locally stable with controllers defined in (2). The agent control u_i is based on state information from other agents to meet federal goals as shown in Fig. 3.

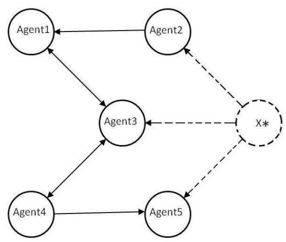

Fig. 3. A Connection Directed Graph of 5 Agents

The corresponding matrix H for a team of five agents with the directed graph is

$$H = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 3 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix} \tag{20}$$

The federated goal is defined as $\dot{x}_* = cos(x_*)$ with the initial condition of 0. The agents are not identical, so their dynamic weights are $a = [0.001; 1; 0.75; 1.5; 1]$. The agents' initial conditions are $x_i = -\pi + \frac{1}{4}\sigma$ in the simulation, where $\sigma = 1, 2, \cdots 5$. The agents' trajectory profile of the first simulation is shown in Fig. 4.

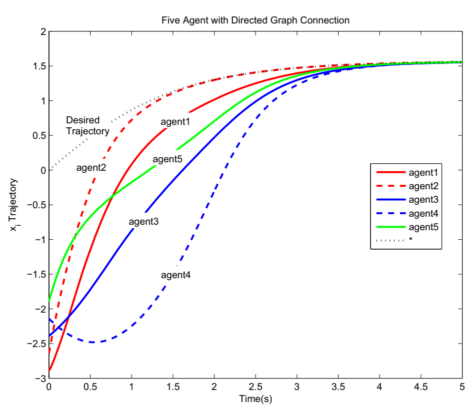

Fig. 4. Trajectory Tracking of 5 Agents with Directed Graph

A second simulation was conducted with an undirected graph and the same initial condition as the previous simulation. The corresponding matrix H is a symmetric matrix for an undirected graph. It implies that H is a positive definite. Therefore, the federated cooperative control with an undirected graph system is a stable system. For the graph shown in Fig. 5, the performance of the system of 5 agents under federated cooperative control (2) is illustrated in Fig. 6.

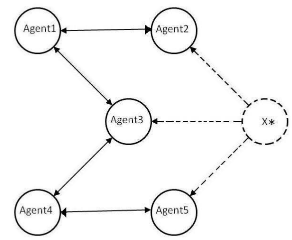

Fig. 5. A Connection Undirected Graph of 5 Agents

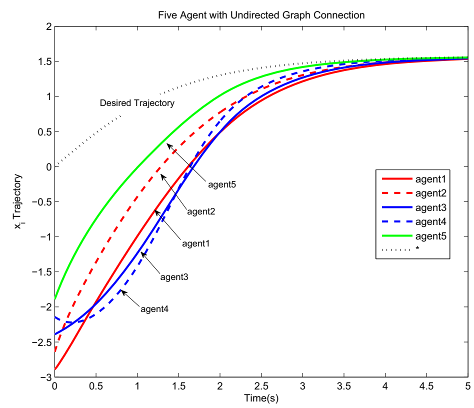

Fig. 6. Trajectory Tracking of 5 Agents with Undirected Graph

Since the control actions for the agents are implemented at the local level, it is possible for the agents to converge to the desired trajectory, as shown in Fig. 4 and Fig. 6. The system consensus is successfully reached under the developed federated control algorithm and the connective strength between agents in the system are optimized.

Moveover, determination of the connective strength between agents plays an important rule in multi-agent federated control. When properly chosen, the strength of the connection between agents (i, j) implies the stability of the multi-agent agent system. In this particular system, as shown in Fig. 4, agents 2, 3 and 5 receive the federated goal. The rest of the agents receive the federated goal through their agent network. Simulation results show that agent 2 has the fastest convergence to the federated trajectory since it has only one dedicated input edge from the federated goal handler. Agent 3 exchanges its information with agents 1 and 4. This bi-directional information exchange holds back the convergence time of agent 3, even though it is directly connected to the federated goal handler. Agent 4 has the slowest convergence time since it only receives information from agent 3. Simulation results of an undirected graph are shown in Fig. 6. In the undirected graph, every node transmits *and* receives information from connected node(s). This multiple information exchange configuration restricts the convergence of all agents to the same time constant. It also slows down the average convergence time compared to a directed graph.

As demonstrated, correct computation of the federated control connective strengths between agents guarantees convergence of every agent in the graph. The graph topology in the directed and undirected graph is the *Consensus Gossip*. A flooding *Consensus Gossip* graph has a sluggish convergence performance. Future work in federated control should include examining system performance under *Consensus Gossip* optimization and analyzing system stability with a dynamical graph.

VI. CONCLUSIONS

This paper presents the concept of federated control of large-scale systems and its implementation using a multi-agent based framework. The multi-agent interconnection strength allows local subsystems to form large-scale systems. Large-scale system stability through a multi-agent based controller is obtained using Lyapunov functions and computing the appropriate agent connection strength. Most importantly, the computation of the appropriate agent connection strength can be executed at the local agent level. Furthermore, we prove that federated control of formation tracking can be achieved as long as the formation graph has a spanning tree and the controller parameters are determined. Simulation results shown that federated control can stabilize multi-agent systems. Finally, the graph Laplacian represents the connectivity among the agents in the system. The developed federated controller can be extended to multi-agent systems with a dynamic graph and improve the system performance under *Consensus Gossip* optimization.

VII. ACKNOWLEDGEMENTS

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